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A universal model of an irreversible combined Carnot cycle system and its general performance characteristics

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Abstract. A universal model of an n-stage combined Carnot cycle system is established. Several major irreversibilities which often exist in real thermodynamic cycles, such as finite-rate heat transfer in the heat-exchange processes, heat leak losses of the heat source, and internal dissipation of the working fluid, are included in the model so that many models of irreversible and endoreversible Carnot cycles which appear in the literature can be regarded as special cases of the universal cycle model. The efficiency, power output and rate of heat input are optimized. Some characteristic curves of the cycle system are presented. Some important performance bounds are given. The optimal combined conditions between two adjacent cycles in the combined cycle system are determined. The optimal performance of an arbitrary-stage irreversible, endoreversible, and reversible combined Carnot cycle system can be directly derived for specific choices of some parameters. The results obtained here are of general significance for both physics and engineering.

1. Introduction

The reversible model of the Carnot cycle has played a major role in the establishment and development of thermodynamics. Almost all thermodynamics texts introduce the Carnot cycles. Although the Carnot efficiency η_c is of great importance in theory, it is invariably far above the efficiency of real heat engines and hence is of very limited practical value. This problem has triggered a series of investigations on the efficiency of heat engines at maximum power output.

Since Rubin [1] defined an endoreversible cycle, which is an extension of reversible cycles, the endoreversible models of the Carnot cycle have been widely used to analyse the performance of heat engines [2–8], refrigerators [9–12], and other thermodynamic systems, such as chemical reactors [13–16], solar cells [17, 18], quantum-mechanical open systems [19, 20]. Besides, other fields such as climatology [21, 22] and computing [23] can also be better described by the endoreversible models of the Carnot cycle. At present the concept of the endoreversible Carnot cycle has appeared in many textbooks [24–28].

Real heat engines are internally reversible. Besides the irreversibility of finite-rate heat transfer across finite temperature differences, there are also other sources of irreversibility, such as heat leaks, turbulence, friction, and the like. In order to analyse the performance of heat engines affected by multi-irreversibilities, some new irreversible cycle models of heat engines have been proposed and many significant results have been obtained [29–34].

Because of the importance of the Carnot cycle in both physics and engineering, it is necessary to develop further new models of irreversible Carnot cycles and investigate

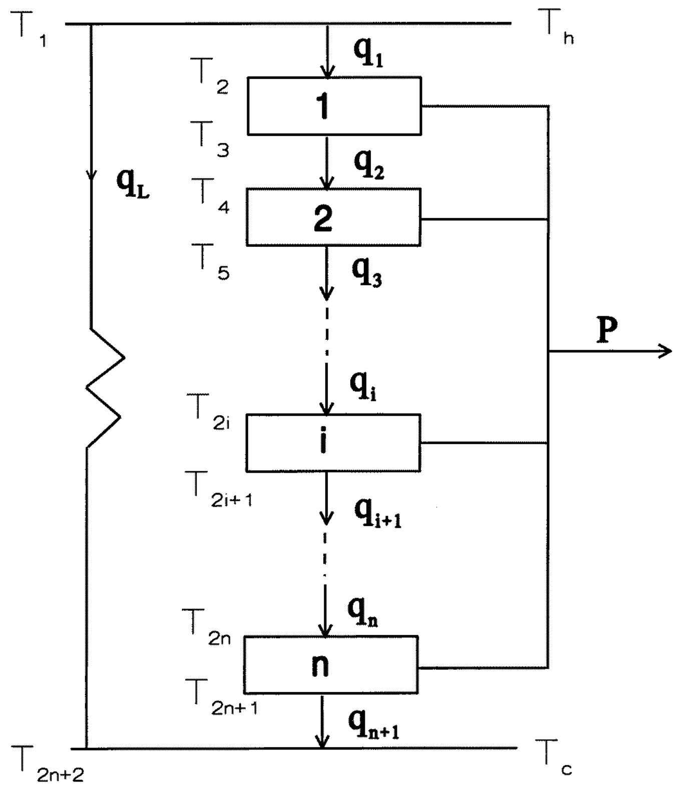

Figure 1. Schematic diagram of an n-stage irreversible combined Carnot cycle system including several major irreversibilities such as finite-rate heat transfer, internal dissipation of the working fluid and heat leak losses of the heat source.

systematically their optimal performance. In this paper, we will establish a universal cycle model and use it to analyse the influence of several major irreversibilities on the performance of an n-stage combined Carnot cycle system.

2. A universal cycle model

Figure 1 shows the schematic diagram of an n-stage irreversible combined Carnot cycle system. Each cycle in the system is connected to the next cycle through the heat exchanger between two adjacent cycles such that the waste heat released from a higher temperature range cycle is used totally as the heat source for the next higher temperature range cycle. The working fluid in the respective cycles flows continuously such that the combined cycle system operates with a steady state. When the effect of the irreversibility of finite-rate heat transfer and the internal dissipation of the working fluid on the performance of the combined cycle system is taken into account, each cycle in the system is an irreversible Carnot cycle and consists of two irreversible isothermal and two irreversible adiabatic processes. In figure 1, q_i and q_{i+1} $(i = 1, 2, \ldots, n)$ are the rates of heat input and output of the ith cycle, T_{2i} and T_{2i+1} $(i = 1, 2, \ldots, n)$ are the temperatures of the working fluid in the high- and low-temperature isothermal processes of the ith cycle, $T_1 = T_h$ and $T_{2n+2} = T_c$ are the temperatures of the heat source and sink, P is the power output of the combined cycle system, and q_L is the heat leak from the heat source to the heat sink [34–36]. Then, the net amounts of heat q_h and q_c released from the heat source and rejected to the heat sink are

$$q_h = q_1 + q_L \tag{1}$$

and

$$q_c = q_{n+1} + q_L \tag{2}$$

respectively.

The performance of an irreversible cycle is directly dependent on the heat transfer law. When heat transfer obeys a Newtonian law, one has

$$q_i = U_i A_i (T_{2i-1} - T_{2i}) \qquad (i = 1, 2, \ldots, n+1) \tag{3}$$

and

$$q_L = k_L (T_h - T_c) \tag{4}$$

where k_L is the heat loss coefficient of the heat source, U_1 and A_i $(i = 2, 3, \ldots, n)$ are the overall heat-transfer coefficient and area of the heat exchanger between the $(i-1)$th and ith cycles, U_1 and A_1 are the overall heat-transfer coefficient and area of the heat exchanger between the heat source at temperature T_h and the first cycle, and U_{n+1} and A_{n+1} are the overall heat-transfer coefficient and area of the heat exchanger between the nth cycle and the heat sink at temperature T_c. The total heat-transfer area of $(n+1)$ heat exchangers in the combined cycle system is

$$A = \sum_{i=1}^{n+1} A_i. \tag{5}$$

According to the cycle model mentioned above and the second law of thermodynamics, we can introduce an irreversibility factor

$$I_i = \frac{q_{i+1}/T_{2i+1}}{q_i/T_{2i}} \geqslant 1 \tag{6}$$

to describe the internal irreversibility of the working fluid in the ith cycle. The total irreversibility due to the internal dissipation of the working fluids in the system may be expressed by

$$I = \prod_{i=1}^{n} I_i. \tag{7}$$

It is clearly seen from equation (6) that when $I_i = 1$, the ith cycle is endoreversible; when $I_i > 1$, the ith cycle is internally irreversible.

The cycle model mentioned above is a universal model of the Carnot cycle. It includes not only the irreversibility of finite-rate heat transfer but also the internal dissipation of the working fluids and the heat leak losses of the heat source. It is important that the optimal performance concerning an arbitrary-stage irreversible, endoreversible, or reversible combined Carnot cycle system may be directly derived from the cycle model.

3. Efficiency and power output

The efficiency and power output are two important parameters of heat engines. Using the above equations, we obtain the efficiency η, power output P, and rate of heat input q_1 of an n-stage combined Carnot cycle system as

$$\begin{aligned}\eta &= \frac{q_h - q_c}{q_h} = \left(1 - \frac{q_{n+1}}{q_1}\right)\frac{q_1}{q_h}\\ &= \left(1 - \frac{IT_3T_5\ldots T_{2n+1}}{T_2T_4\ldots T_{2n}}\right)\frac{q_1}{q_1+q_L}\\ &= \left(1 - \frac{IT_c/T_h}{x_1x_2\ldots x_{n+1}}\right)\frac{q_1}{q_1+q_L}\end{aligned} \tag{8}$$

$$P = q_h - q_c = (q_1 + q_L)\eta \tag{9}$$

and

$$\begin{aligned}q_1 &= A\frac{q_1}{\sum_{i=1}^{n+1} A_i} = \frac{A}{\sum_{i=1}^{n+1}\frac{q_i}{U_i(T_{2i-1}-T_{2i})q_1}}\\ &= \frac{A}{\frac{1}{U_1(T_1-T_2)} + \frac{I_1T_3/T_2}{U_2(T_3-T_4)} + \frac{I_1I_2T_3T_5/(T_2T_4)}{U_3(T_5-T_6)} + \cdots + \frac{IT_3T_5\ldots T_{2n+1}}{U_{n+1}(T_{2n+1}-T_{2n+2})T_2T_4\ldots T_{2n}}}\\ &= \frac{AT_h}{\frac{1}{U_1^*(1-x_1)} + \frac{1}{U_2^*x_1(1-x_2)} + \frac{1}{U_3^*x_1x_2(1-x_3)} + \cdots + \frac{1}{U_{n+1}^*x_1x_2\ldots x_n(1-x_{n+1})}}\end{aligned} \tag{10}$$

where $x_i = T_{2i}/T_{2i-1} (i = 1, 2, \ldots, n+1)$ and

$$U_i^* = U_i/\prod_{j=0}^{i-1} I_j \qquad (i = 1, 2, 3, \ldots, n+1) \tag{11}$$

is the equivalent overall heat-transfer coefficient of the heat exchanger between the $(i-1)$th and ith cycles. The I_0 appearing in equation (11) is stipulated to be equal to 1.

Eliminating x_{n+1} in equations (8) and (10) gives

$$\eta = \left(1 - \frac{IT_c/T_h}{x_1x_2\ldots x_n - \frac{1}{U_{n+1}^*D}}\right)\frac{q_1}{q_1+q_L} \tag{12}$$

where

$$\begin{aligned}D = \frac{AT_h}{q_1} - \frac{1}{U_1^*(1-x_1)} - \frac{1}{U_2^*x_1(1-x_2)} - \frac{1}{U_3^*x_1x_2(1-x_3)} - \cdots\\ - \frac{1}{U_n^*x_1x_2\ldots x_{n-1}(1-x_n)}.\end{aligned} \tag{13}$$

4. Characteristic curves

For a given rate of heat input q_1 and total heat-exchange area A, using equation (12) and the extremal conditions

$$\frac{\partial\eta}{\partial x_i} = 0 \qquad (i = 1, 2, \ldots, n) \tag{14}$$

we find that the efficiency is given by (a detailed derivation is given in appendix A)

$$\eta = \left(1 - \frac{IT_c/T_h}{1-q_1^*}\right)\frac{q_1^*}{q_1^*+C} \tag{15}$$

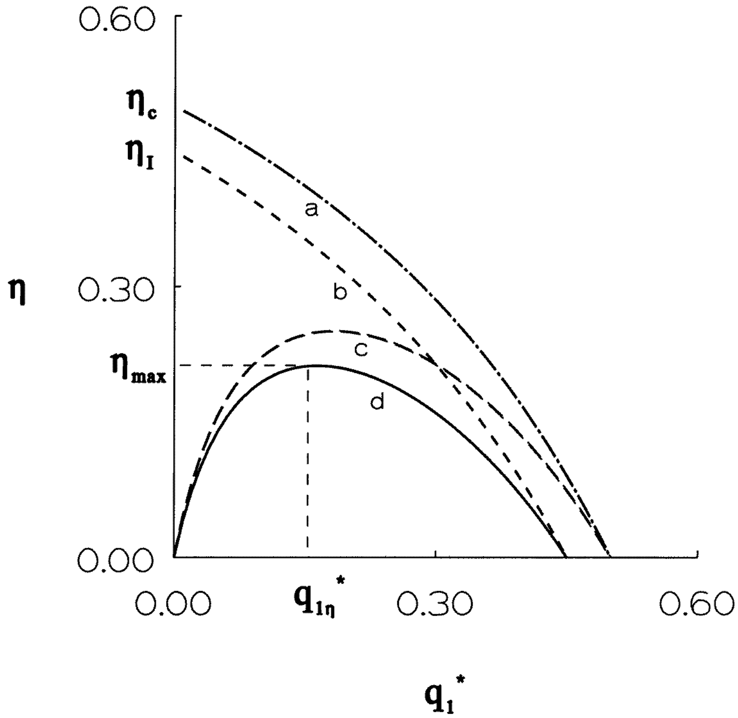

Figure 2. Efficiency η versus dimensionless rate of heat input q_1^*. Curves: a ($I = 1$, $C = 0$), b ($I = 1.1$, $C = 0$), c ($I = 1$, $C = 0.1$), and d ($I = 1.1$, $C = 0.1$) are presented for $T_c/T_h = 0.5$.

where $q_1^* = q_1/(U^*AT_h)$, $C = q_L/(U^*AT_h)$, and

$$U^* = 1\bigg/\left(\sum_{i=1}^{n+1}\frac{1}{\sqrt{U_i^*}}\right)^2 \tag{16}$$

is the equivalent overall heat-transfer coefficient of the combined cycle system.

Substituting equation (15) into equation (9) gives the relation between the dimensionless power output $P^* = P/(U^*AT_h)$ and rate of heat input q_1^* as

$$P^* = \left(1 - \frac{IT_c/T_h}{1-q_1^*}\right)q_1^*. \tag{17}$$

From equations (15) and (17), we obtain the η–q_1^* and P^*–q_1^* characteristic curves, as shown in figures 2 and 3, respectively. It is seen from figures 2 and 3 that although the power output is not affected by the heat leak losses of the heat source, the efficiency of the cycle system is obviously dependent on the heat leak losses of the heat source.

Eliminating q_1^* in equations (15) and (17), we obtain the optimal relation between the dimensionless power output P^* and the efficiency η as

$$(1-\eta)(P^*)^2 - [\eta_I + 2C - (1+C)\eta]\eta P^* + (C+\eta_I)C\eta^2 = 0 \tag{18}$$

where $\eta_I = 1 - IT_c/T_h$.

Using equation (18), we can generate the P^*–η characteristic curves of an n-stage combined Carnot cycle system, as shown in figure 4.

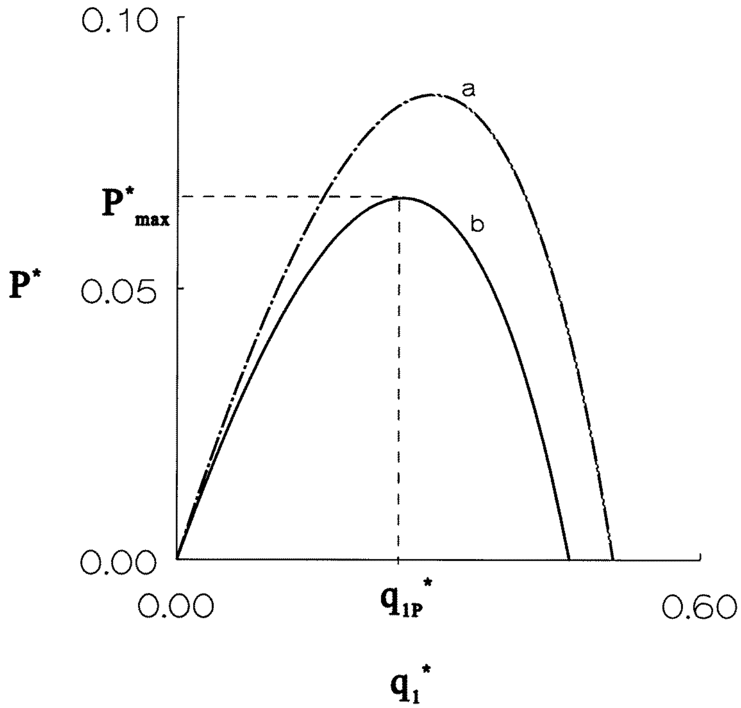

Figure 3. The dimensionless power output P^* versus rate of heat input q_1^*. Curves: a ($I = 1$) and b ($I = 1.1$) are presented for $T_c/T_h = 0.5$.

5. Some important performance bounds

Figure 4 shows clearly that when the heat leak losses of the heat source is taken into account, the P^*–η characteristic curves of an n-stage combined Carnot cycle system are of loop shapes. There exist a maximum power output and a maximum efficiency.

From equation (17) and its extremal condition

$$\frac{\mathrm{d}P^*}{\mathrm{d}q_1^*} = 0 \tag{19}$$

we can prove that when

$$q_1^* = 1 - \sqrt{1 - \eta_I} \equiv q_{1P}^* \tag{20}$$

the dimensionless power output of the cycle system attains its maximum, i.e.

$$P^*_{\max} = \left(1 - \sqrt{1 - \eta_I}\right)^2 \tag{21}$$

and the corresponding efficiency is given by

$$\eta_m = \frac{\left(1 - \sqrt{1 - \eta_I}\right)^2}{1 + C - \sqrt{1 - \eta_I}}. \tag{22}$$

Incidentally, equations (21) and (22) can also be derived from equation (18) (a detailed derivation is given in appendix B).

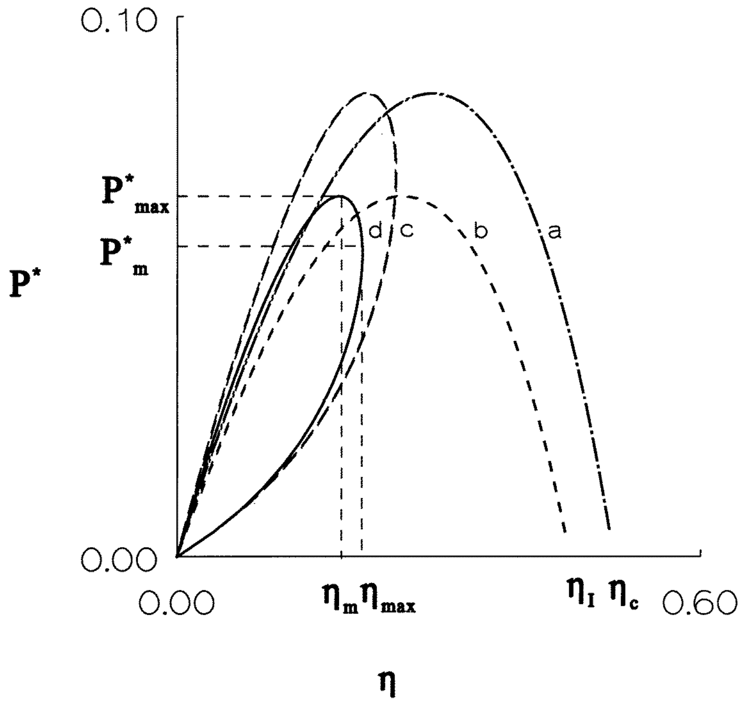

Figure 4. The dimensionless power output P^* versus efficiency η. The values of I, C, and T_c/T_h are the same as those adopted in figure 2.

Using equation (18) and the similar method mentioned in appendix B, we can prove that the maximum efficiency of the cycle system is determined by the following equation

$$[\eta_I + 2C - (1+C)\eta_{\max}]^2 - 4(C+\eta_I)C(1-\eta_{\max}) = 0 \tag{23}$$

and the corresponding dimensionless power output is given by

$$P_m^* = \frac{[\eta_I + 2C - (1+C)\eta_{\max}]\eta_{\max}}{2(1-\eta_{\max})}. \tag{24}$$

Solving equations (23) and (24), we obtain

$$\eta_{\max} = \frac{\left[\sqrt{C+\eta_I} - \sqrt{C(1-\eta_I)}\right]^2}{(1+C)^2} \tag{25}$$

and

$$P_m^* = \frac{\sqrt{C+\eta_I}\left[\sqrt{C+\eta_I} - \sqrt{C(1-\eta_I)}\right]^2}{(1+C)\left[\sqrt{C+\eta_I} + \sqrt{(1-\eta_I)/C}\right]}. \tag{26}$$

From equations (24), (25), and (9), we obtain the dimensionless rate of heat input at maximum efficiency as

$$q_1^* = \frac{\sqrt{C(C+\eta_I)(1-\eta_I)} - C}{1-\eta_I - C} \equiv q_{1\eta}^*. \tag{27}$$

Of course, equations (25)–(27) can also be directly derived from equation (15) and its extremal condition.

It is clearly seen from figure 4 that when $\eta < \eta_m$ or $P^* < P^*_m$, the power output decreases as the efficiency decreases. These regions are not the optimal regions of the heat engines. The optimal regions should be situated in the parts of the P^*–η curves with negative slope. When the heat engines operate in these working states, the power output will increase as the efficiency decreases, and vice versa. Therefore, the dimensionless power output P^* and efficiency η must be constrained by

$$\eta_m \leqslant \eta \leqslant \eta_{\max} \tag{28}$$

and

$$P^*_{\max} \geqslant P^* \geqslant P^*_m. \tag{29}$$

This shows that $\eta_{\max}$, η_m, $P^*_{\max}$ and P^*_m are the four important performance parameters of Carnot cycle systems. They determine the upper bounds and the allowable values of the lower bounds of the efficiency and the dimensionless power output, respectively. According to equation (28) or (29), we can further determine that the optimal range of the dimensionless rate of heat input must be constrained by

$$q^*_{1P} \geqslant q^*_1 \geqslant q^*_{1\eta}. \tag{30}$$

6. Optimal combined conditions

In order to make the n-stage combined Carnot cycle system operate in the optimum working states, the parameters in the system must satisfy certain conditions.

Using equations (3), (6), and (A11), we obtain the optimal relation of the heat-exchange areas

$$\sqrt{U^*_i}A_i = \sqrt{U^*_{i+1}}A_{i+1} \qquad (i = 1, 2, \ldots, n). \tag{31}$$

Solving equation (31) and (5) gives the optimal distribution of the heat-transfer areas of $(n+1)$ heat exchangers in the combined cycle system

$$A_i = \frac{A}{\sum_{j=1}^{n+1}\sqrt{U^*_i/U^*_j}} \qquad (i = 1, 2, \ldots, n+1). \tag{32}$$

From equations (A11), (A12), (15), and (12), we obtain

$$x_1 = 1 - \frac{q^*_1}{\sum_{j=1}^{n+1}\sqrt{U^*_1/U^*_j}} \tag{33}$$

$$x_i = 1 - \frac{q^*_1}{\sum_{j=1}^{n+1}\sqrt{U^*_i/U^*_j}\,x_1x_2\ldots x_{i-1}} \qquad (i = 2, 3, \ldots, n) \tag{34}$$

and

$$x_{n+1} = \frac{1-q^*_1}{x_1x_2\ldots x_n}. \tag{35}$$

Substituting equations (20) and (27) into equations (33)–(35), we obtain respectively the temperature ratios x_{iP} and $x_{i\eta}$ in the heat-transfer processes at maximum power output and maximum efficiency. According to equation (28) or (29), the optimal range of the temperature ratios x_i in the heat transfer processes must be situated in the region between x_{iP} and $x_{i\eta}$.

So far we have determined the optimal combined conditions of an n-stage combined Carnot cycle system operating in the optimum working states.

7. Several special cases

(i) When $I_i = 1$ $(i = 1, 2, \ldots, n)$, there does not exist the internal dissipation of the working fluid and the Carnot cycle is endoreversible. In such a case,

$$U^* = 1 \Bigg/ \left(\sum_{i=1}^{n+1} \frac{1}{\sqrt{U_i}} \right)^2 = U \tag{36}$$

and equation (18) may be written as

$$(1-\eta)(P^*)^2 - [\eta_c + 2C - (1+C)\eta]\eta P^* + (C+\eta_c)C\eta^2 = 0. \tag{37}$$

The P^*–η characteristic curve is given by curve c in figure 4. Equation (37) can be used to discuss the optimal performance of an arbitrary-stage endoreversible combined Carnot cycle system.

(ii) When $U_i \to \infty$ $(i = 2, 3, \ldots, n)$, the irreversibility of finite-rate heat transfer between two adjacent cycles in the combined cycle system is negligible. The equivalent overall heat-transfer coefficient of the n-stage irreversible combined Carnot cycle system may be simplified as

$$U^* = \frac{1}{\left(1/\sqrt{U_1} + 1/\sqrt{U_{n+1}/I}\right)^2} \tag{38}$$

which is identical to that of a single-stage irreversible Carnot cycle having the same irreversibility factor I, so that equation (18) is the same as the relation between the power output and the efficiency of a single-stage irreversible Carnot cycle.

(iii) When $k_L = 0$, equation (18) may be written as

$$P^* = \eta \frac{\eta_I - \eta}{1-\eta}. \tag{39}$$

The P^*–η characteristic curve is not of loop shape, which is shown in curve b in figure 4. The dimensionless maximum power output is still given by equation (21), while the corresponding efficiency becomes [37]

$$\eta_m = 1 - \sqrt{1-\eta_I}. \tag{40}$$

(iv) When $k_L = 0$ and $I_i = 1$ $(i = 1, 2, \ldots, n)$, $U^* = U$ and equation (39) becomes

$$P^* = \eta \frac{\eta_c - \eta}{1-\eta} \tag{41}$$

which is identical to the P^*–η relation of an n-stage combined endoreversible Carnot cycle system [38, 7]. The efficiency at maximum power output is given by

$$\eta_m = 1 - \sqrt{T_c/T_h} \tag{42}$$

which was independently found by several authors [39–41] and has widely appeared in the literature [1, 3, 5–7, 34, 36–38, 42] and textbooks [24–28].

(v) When $k_L = 0$, $I_i = 1$ $(i = 1, 2, \ldots, n)$, and $U_i \to \infty$ $(i = 1, 2, \ldots, n+1)$, the Carnot cycle system is reversible. It can be derived from equation (18) that the efficiency

$$\eta = \eta_c = 1 - T_c/T_h \tag{43}$$

is only the temperatures of the heat source and sink and independent of both the rate of heat input q_1 and the power output P. In such a case, the power output

$$P = q_1 \eta_c \tag{44}$$

may be an arbitrary value, which is determined by the rate of the heat input q_1. This shows that the results obtained in this paper are also suitable for reversible Carnot cycles.

8. Conclusions

A universal model of an n-stage combined Carnot cycle system established in this paper is the development of the endoreversible and reversible Carnot cycle models. The universal cycle model includes finite-rate heat transfer, heat leak, and internal dissipation of the working fluid so that it can capture the principal irreversibility sources of some real heat engines. New bounds of the key performance parameters in the combined cycle system are determined. It is important that the results derived from this universal cycle model can be suitable for an arbitrary-stage irreversible, endoreversible, or reversible combined Carnot cycle system and, consequently, reveal some common characteristics of Carnot cycle systems. It is more important that this universal cycle model established in this paper is of general significance for both physics and energy engineering and may promote the further investigation of irreversible Carnot cycles and their applications in other fields.

Acknowledgments

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Appendix A

From equations (12) and (14), we obtain

$$U_{n+1}^* D^2 x_2 x_3 \ldots x_n - \frac{1}{U_1^*(1-x_1)^2} + \frac{1}{U_2^* x_1^2(1-x_2)} + \frac{1}{U_3^* x_1^2 x_2 (1-x_3)} + \cdots + \frac{1}{U_n^* x_1^2 x_2 \ldots x_{n-1}(1-x_n)} = 0 \tag{A1}$$

$$U_{n+1}^* D^2 x_1 x_3 \ldots x_n - \frac{1}{U_2^* x_1 (1-x_2)^2} + \frac{1}{U_3^* x_1 x_2^2 (1-x_3)} + \cdots + \frac{1}{U_n^* x_1 x_2^2 x_3 \ldots x_{n-1}(1-x_n)} = 0 \tag{A2}$$

...

$$U_{n+1}^* D^2 x_1 x_2 \ldots x_{i-1} x_{i+1} \ldots x_n - \frac{1}{U_i^* x_1 x_2 \ldots x_{i-1}(1-x_i)^2} + \cdots + \frac{1}{U_n^* x_1 x_2 \ldots x_{i-1} x_i^2 x_{i+1} \ldots x_{n-1}(1-x_n)} = 0 \tag{A3}$$

$$U_{n+1}^* D^2 x_1 x_2 \ldots x_{n-2} x_n - \frac{1}{U_{n-1}^* x_1 x_2 \ldots x_{n-2}(1-x_{n-1})^2} + \cdots + \frac{1}{U_n^* x_1 x_2 \ldots x_{n-2} x_{n-1}^2 (1-x_n)} = 0 \tag{A4}$$

and

$$U_{n+1}^* D^2 x_1 x_2 \ldots x_{n-1} - \frac{1}{U_n^* x_1 x_2 \ldots x_{n-1}(1-x_n)^2} = 0. \tag{A5}$$

Solving equations (A1)–(A5), we have

$$x_1(1-x_2) = \sqrt{U_1^*/U_2^*}(1-x_1) \tag{A6}$$

$$x_2(1-x_3)=\sqrt{U_2^*/U_3^*}(1-x_2) \tag{A7}$$

...

$$x_i(1-x_{i+1})=\sqrt{U_i^*/U_{i+1}^*}(1-x_i) \tag{A8}$$

...

$$x_{n-1}(1-x_n)=\sqrt{U_{n-1}^*/U_n^*}(1-x_{n-1}) \tag{A9}$$

and

$$(1-x_n)=\sqrt{\frac{1}{U_n^*U_{n-1}^*}\frac{1}{Dx_1x_2\ldots x_{n-1}}}. \tag{A10}$$

From equations (A6)–(A10), we can derive some optimal relations as

$$\begin{aligned}\sqrt{U_1^*}(1-x_1)&=\sqrt{U_2^*}x_1(1-x_2)=\sqrt{U_3^*}x_1x_2(1-x_3)=\cdots\\ &=\sqrt{U_i^*}x_1x_2\ldots x_{i-1}(1-x_i)=\cdots=\sqrt{U_n^*}x_1x_2\ldots x_{n-1}(1-x_n)\\ &=\frac{\sqrt{1/U_{n+1}^*}+\sqrt{1/U_n^*}}{\frac{AT_h}{q_1}-\frac{1}{U_1(1-x_1)}-\frac{1}{U_2^*x_1(1-x_2)}-\cdots-\frac{1}{U_{n-1}^*x_1x_2\ldots x_{n-2}(1-x_{n-1})}}.\end{aligned} \tag{A11}$$

Solving equation (A11), we obtain

$$\sqrt{U_1^*}(1-x_1)=\frac{q_1}{AT_h}\sum_{i=1}^{n+1}\frac{1}{\sqrt{U_i^*}} \tag{A12}$$

and

$$x_1x_2\ldots x_n=1-\frac{1}{D\sqrt{U_{n+1}^*}}\sum_{i=1}^{n}\frac{1}{\sqrt{U_i^*}}. \tag{A13}$$

Substituting equations (A11)–(A13) into equation (12), we obtain equation (15).

Appendix B

In order to determine the maximum power output of the combined cycle system, equation (18) may be written as a quadratic equation of the efficiency η, i.e.

$$a\eta^2+b\eta+c=0 \tag{B1}$$

where

$$\begin{aligned}a&=C(C+\eta_I)+(1+C)P^*\\ b&=-(2C+\eta_I+P^*)P^*\\ c&=(P^*)^2.\end{aligned} \tag{B2}$$

It can be proven from equation (B1) that the dimensionless maximum power output of the cycle system is determined by the following equation

$$b^2-4ac=0 \tag{B3}$$

and the corresponding efficiency is given by

$$\eta_m=-b/(2a). \tag{B4}$$

Substituting equation (B2) into equations (B3) and (B4), we obtain

$$(2C+\eta_I+P_{\max}^*)^2-4[C(C+\eta_I)+(1+C)P_{\max}^*]=0 \tag{B5}$$

and

$$\eta_m = \frac{(2C + \eta_I + P^*_{\max})P^*_{\max}}{2[C(C + \eta_I) + (1 + C)P^*_{\max}]}. \tag{B6}$$

Solving equations (B5) and (B6), we obtain equations (21) and (22).

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